

HOLOGRAPHIC INTERFEROMETRY MEASUREMENT OF CONVECTIVE HEAT TRANSPORT BENEATH A HEATED HORIZONTAL PLATE IN AIR

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Abstract—The technique of live-fringe holographic interferometry has been used in an investigation of convective heat transfer beneath a heated horizontal plate in air. Two copper plates were used, one a 5.1 cm square, the other a 16 cm square. Temperature profiles were measured in air beneath isothermal plate surfaces at Rayleigh numbers ranging from 4×10^4 to 1.5×10^6 . The paper describes apparatus and procedures used in the holographic process as well as methods used in deriving temperature profiles from interferograms. Measured temperature profiles were in excellent agreement with predictions based on approximate analytical solutions to the heat transfer problem previously reported by others. Plate-average Nusselt numbers, derived from the measured temperature profiles, exceeded predicted values but were in accord with previous measurements.

NOMENCLATURE

a ,	plate characteristic dimension [m];
b ,	plate dimension [m];
g ,	acceleration of gravity [$m\ s^{-2}$];
K ,	Gladstone-Dale constant [$m^3\ kg^{-1}$];
n ,	refractive index;
Nu ,	Nusselt number;
p ,	pressure [Pa];
R ,	ideal gas constant [$m^3\ Pa\ kg^{-1}\ K^{-1}$];
Ra ,	Rayleigh number;
T ,	temperature [K];
v ,	$z/\delta(0, y)$;
X, Y, Z ,	spatial coordinates;
x, y, z ,	$X/a, Y/a, Z/a$;
α ,	thermal diffusivity [$m^2\ s^{-1}$];
β ,	coefficient of thermal expansion [K^{-1}];
δ ,	dimensionless boundary layer thickness ($2/Nu$);
ϵ ,	fringe displacement;
θ ,	$T - T_\infty$ [K];
λ ,	wavelength [m];
ν ,	kinematic viscosity [$m^2\ s^{-1}$];
ξ ,	$(1 - x^2)(1 - y^2)$;
ρ ,	density [$kg\ m^{-3}$];
ϕ ,	$(T - T_\infty)/(T_0 - T_\infty)$.

Subscripts

0,	conditions at heat transfer plate ($z = 0$);
∞ ,	ambient conditions.

1. INTRODUCTION

MEASUREMENTS have been made of convective heat

transport in air beneath the surface of a horizontal square plate held at a constant temperature above that of the surroundings. Two plates were used, both made of copper, one a 5.1 cm square, the other a 16 cm square. Plate temperatures varied from 10 to 50 K above the ambient temperature. Live-fringe holographic interferometry was used to measure temperature profiles from which were determined convective heat transfer coefficients and boundary layer thicknesses. Results were correlated in terms of the Nusselt number vs the Rayleigh number.

Figure 1 establishes the geometrical notation used here. Within the fluid, the dimensionless temperature index is

$$\phi(x, y, z) = \theta(x, y, z)/\theta_0 \quad (1)$$

in which θ_0 is the plate temperature T_0 less the ambient temperature T_∞ , and $\theta(x, y, z)$ is the local fluid temperature $T(x, y, z)$ less the ambient temperature. Dimensionless position indices are defined in terms of

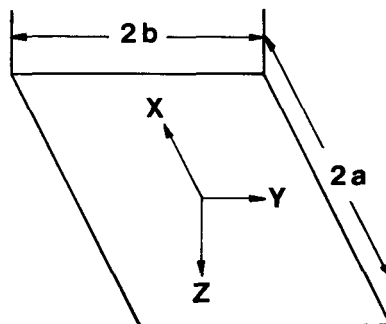


FIG. 1. Geometry for the downward heat transfer experiment. The plate surface temperature is T_0 . Ambient temperature is T_∞ .

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the characteristic plate dimension, viz. $x = X/a$. The Rayleigh and Nusselt numbers, defined in terms of the same characteristic length, are respectively

$$Ra = g\beta a^3 \theta_0 / \alpha \nu \tag{2}$$

$$Nu(x, y) = - \frac{\partial \phi}{\partial z}(x, y, 0). \tag{3}$$

Average Nusselt numbers of interest are

$$\overline{Nu}(y) = \int_0^1 dx Nu(x, y) \tag{4}$$

and

$$\overline{\overline{Nu}} = \int_0^1 dy \int_0^1 dx Nu(x, y). \tag{5}$$

A dimensionless index of the boundary layer thickness may be defined as

$$\delta(x, y) = 2/Nu(x, y). \tag{6}$$

Similarly,

$$\overline{\delta}(y) = 2/\overline{Nu}(y). \tag{7}$$

Experimental studies of downward heat transfer by convection from horizontal surfaces have built upon the early efforts of Weise [1], Saunders, Fishenden and Mansion [2] and Kraus [3], whose investigations had established that, over a wide range of parameters, the Nusselt number for heat transfer was proportional to the 0.2 power of the Rayleigh number, and only weakly dependent on the surface shape and edge conditions. Recent experimental studies include the work of Fujii and Imura [4] and Aihara, Yamada and Endo [5]. The former measured average Nusselt numbers for heat transfer to water from rectangular brass plates $5 \times 10 \times 0.5$ cm thick and $15 \times 30 \times 1$ cm thick. In an effort to establish two-dimensional flow patterns, the plate sides were insulated along the shorter dimensions ($Y = \pm b$ in Fig. 1). The uninsulated sides were presumably at or near the plate surface temperature. Over the range $10^6 \leq Ra \leq 10^{11}$ boundary layer flow was found to be laminar, with

$$\overline{Nu}(y) = 0.58 Ra^{0.2}. \tag{8}$$

Note that this is an average over the X -coordinate, with the b -dimension assumed to be infinite.

Aihara and his co-workers studied heat transfer to air from a brass plate $25 \times 35 \times 1$ cm thick. The shorter sides of the plate were insulated and the uninsulated sides were presumably at or near the plate surface temperature. With reference to Fig. 1, a thermocouple probe was used to measure $\phi(0, y, z)$ for $0 \leq y \leq 1$. Except for $y > 0.8$, observed temperature profiles were well represented by the equation

$$\phi(0, y, z) = [1 + z/\delta(0, y)][1 - z/\delta(0, y)]^3. \tag{9}$$

Temperature profiles and local Nusselt numbers were in substantial agreement with those predicted by Singh and Birkebak [6] for an infinite strip (see below).

Birkebak and Abdulkadir [7] investigated heat transfer to water from an aluminium plate 19 cm square and 1.25 cm thick. By a special cooling system, plate sides were held at ambient temperature. Using a thermocouple probe, values of $\phi(0, y, z)$ were measured for $y = 0, 0.5$ and 0.85 . Except near the edges of the plate, temperature profiles and local Nusselt numbers were in agreement with predictions of Singh, Birkebak and Drake [8] for a square plate (see below). In agreement with previous observations, [1-3, 5] the boundary-layer thicknesses were found to be finite at the plate edges. The average Nusselt number over the entire plate surface was found to be

$$\overline{\overline{Nu}} = 0.68 Ra^{0.2}. \tag{10}$$

Restrepo and Glicksman [9] studied heat transfer to air from a copper plate $7 \times 7 \times 1$ in. thick with side surfaces: (1) insulated; (2) heated to the plate surface temperature; or (3) cooled to ambient temperature. Temperature profiles $\phi(0, y, z)$ were measured using a thermocouple probe. At the plate edges, boundary-layer thicknesses were much greater for the cooled sides than for the heated sides and for the former were as thick as $\frac{1}{8}$ the plate dimensions. For plates with heated sides, plate-average Nusselt numbers agreed with equation (8).

Analytical studies of downward convection from plates have been carried out using the boundary-layer approximation. Stewartson [10] and Gill, Zeh and del Casal [11] obtained similarity solutions for a two-dimensional case. Integral methods were applied by Levy [12] and Wagner [13] for the semi-infinite strip. The method was extended by Singh, Birkebak and Drake [8] to rectangular and circular plates. These analyses involved assuming the thermal boundary layer to vanish at the plate edge. For the special case of the infinite strip, Singh and Birkebak [6] were able to obtain solutions accommodating finite boundary-layer thicknesses at the plate edge.

For the rectangular plate, the subject of this experimental study, the analysis of Singh, Birkebak and Drake [8] leads to temperature profiles of the form

$$\phi(x, y, z) = \left[1 - \frac{z}{\delta(x, y)}\right]^2 \tag{11}$$

in which, to a first-order approximation

$$\delta(x, y) = \delta_0 \xi^{1/4}(x, y) \tag{12}$$

with

$$\delta_0 = \delta(0, 0) = 4.357 Ra^{-1/5} \tag{13}$$

and

$$\xi(x, y) = (1 - x^2)(1 - y^2). \tag{14}$$

The local Nusselt number is thus

$$Nu(x, y) = 2/\delta(x, y) = 0.459 \xi^{-1/4}(x, y) Ra^{1/5}. \tag{15}$$

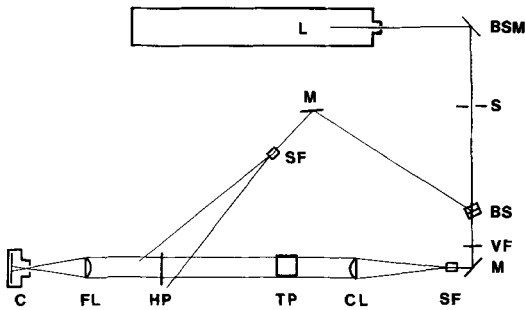


FIG. 2. Experimental arrangement for holographic interferometry. L = laser, BSM = beam positioning mirrors, S = shutter, BS = beam splitter, M = mirror, VF = variable filter, SF = spatial filter, CL = collimating lens, TP = heat transfer test plate, HP = hologram plate, FL = focusing lens, C = camera.

2. EXPERIMENTAL PROCEDURE

Measurements were made of temperature profiles in air beneath the surface of a heated, downward-facing, horizontal plate using live-fringe holographic interferometry. A general discussion of the technique and apparatus has been described elsewhere [14]. Using an argon-ion laser, a hologram was made of the plate and its surroundings, all at ambient temperature. The plate temperature was then increased, while the hologram and plate were illuminated by the laser light under conditions otherwise identical to those during the preparation of the hologram. The holographic image was superimposed on the real plate and surroundings. Changes in refractive index of the heated air led to optical interference patterns which were recorded photographically. From the interference patterns were determined temperature profiles.* With reference to Fig. 1, the temperatures measured were averages over the x -coordinate, i.e.

$$\bar{\phi}(y, z) = \int_0^1 dx \phi(x, y, z). \quad (16)$$

From the temperature profiles were derived the average Nusselt numbers $\overline{Nu}(y)$ and \overline{Nu} .

The overall arrangement of apparatus is illustrated in Fig. 2. Apparatus was positioned on a granite bench, 4 ft \times 10 ft \times 4 in. thick. Isolation from vibration was provided by supporting the bench at each corner on lightly inflated rubber-tyre inner tubes. The tubes rested on sand-filled concrete cylinders. The cylinders rested on a sand base in larger concentric concrete cylinders, also sand-filled. (For further details, see [14].)

The beam from an argon-ion laser was elevated to 26 cm above the bench by a pair of beam positioning mirrors. After passing through a shutter, the beam was divided into two equal components by a beam-splitting cube. One beam, the reference beam, was expanded by a 20 \times microscope objective and filtered by an 11 μ m diameter pin-hole. The objective was located 85 cm from the hologram plate and the expanded beam struck the plate at an angle of about 45°. A second beam, the object beam, was expanded and filtered by a 20 \times objective and 11 μ m pin-hole lens located 38 cm from the centre of the heat transfer plate. A variable density filter was used to adjust the intensity of the object beam. The X -axis of the heat-transfer plate (Fig. 1) was aligned with the axis of the object beam.

The laser used was a high-power, argon-ion laser, Spectra-Physics Model 171-06. It had a nominal power rating of 9 W (multi-line) and 3.75 W (514.5 nm-line). Available power was considerably in excess of rated power. Single-frequency operation was made possible by the use of an etalon. Spectra-Physics Model 589, in the laser cavity. With the etalon in use, available power at 514.5 nm was about 2.5 W. Laser beam diameter (e^{-2} points) was nominally 1.6 mm with a full-angle beam divergence of 0.72 mrad.

Mirrors, beam expanders, positioners, splitters, and spatial filters were components distributed by Ealing-Beck, Ltd. For high-power operation, uncemented microscope objectives, manufactured by W. R. Prior, were used. The reference-beam mirror was a 4 in. diameter Schlieren-quality mirror made by Optical Works Ltd. The hologram plate holder was a Micro-Control, Type SH37 Plate Support distributed by Unimatic Engineers Ltd. Hologram plates were 8E56 (AHI) Holotest Plates made on special order by Agfa-Gevaert Ltd. The plates were 4 in. \times 5 in. with thickness 1.5–1.7 mm. Plate flatness is 10 μ m/cm or better.

Two heat-transfer plates were used. One was a square copper plate, 16 cm on a side and 1.25 cm thick. Thermocouple wells, 1.25 mm diameter, were drilled to 1 mm from the plate surface. Sixteen wells were located within one quarter-sector of the plate. The plate was heated by four independent nichrome heating elements encased in ceramic insulating tubes recessed into grooves in an insulating backing plate (resin-impregnated fibre-board). The backing plate was square, 15.8 cm on a side, and was slightly smaller than the heat-transfer plate, so as not to disturb flow around the edge of the heat-transfer plate. The heating elements were in an array of concentric squares spaced to provide approximately uniform heating. The elements were individually powered through variable transformers connected to 240 V, 50 Hz mains power. To facilitate adjustment, variable resistors (22 ohm, 10 A rating) were placed in series with the heating elements. With these arrangements, the plate temperature was easily held uniform to within 0.5 K. The second plate

* As discussed in Section 3, interference patterns yield a harmonic average temperature, closely approximated by equation (16).

was also square, 5.1 cm long and 1 cm thick, backed by a 5 cm square insulating plate. Nichrome heating elements, in an array of three concentric squares, were placed in ceramic insulating tubes and positioned in recessed grooves in the backing plate. The elements were connected in series. Plate temperature was monitored by three thermocouples placed in 1.25 mm diameter wells drilled in the copper plate to 0.5 mm from the downward-facing surface.

The test plate was levelled and aligned using the horizontal unexpanded object beam from the laser. The laser beam was also used to align the camera used to photograph interference patterns. With the alignment beam along the centre-line of the plate, the levelled camera was positioned so that the beam passed through the aperture at its smallest stop and through the centre of the film-holder. To minimize effects of air currents, the test plate was surrounded on four sides by a glass-windowed enclosure made of heavy cardboard. The enclosure was open at the top and had air entrance slits at its base.

With the heat-transfer plate at ambient temperature, a hologram plate was exposed to an incident energy density of about $5 \mu\text{J cm}^{-2}$, with the intensity of the reference beam twice that of the object beam. The hologram plate was processed using a procedure similar to that described by Phillips and Porter [15].

The essential features were development in indiluted Neofin Blau (Tetenal Photowerk), and bleaching in an aqueous solution of ferric nitrate, potassium bromide and isopropanol.

With the heat-transfer test plate still at ambient temperature, the hologram plate was carefully repositioned in its holder and illuminated by the object and reference beams. A virtual holographic image of the test plate then appeared superimposed on the actual plate. The hologram plate was then displaced slightly to produce a system of parallel interference fringes (see Fig. 4), the so-called finite-fringe setting. Power was then applied to the heating elements of the test plate. The plate temperature was increased at a rate of about 1 K per minute while carefully monitoring the plate temperature to assure uniformity. Convective heat transfer from the plate caused density changes and corresponding refractive index changes in the air. These changes caused optical interference leading to distortion of the fringes.

Interference patterns were photographed on microfilm (Eastman Kodak 5669 Micro-File) using a 35 mm single lens reflex camera (Asahi Pentax KX) with a 50 mm focal length $f/1.7$ lens, augmented by a 15 cm diameter, 60 cm focal length lens (Tropel 280-150). Typical exposure time was $1/250$ s with the laser power at 50 mW. The microfilm was processed

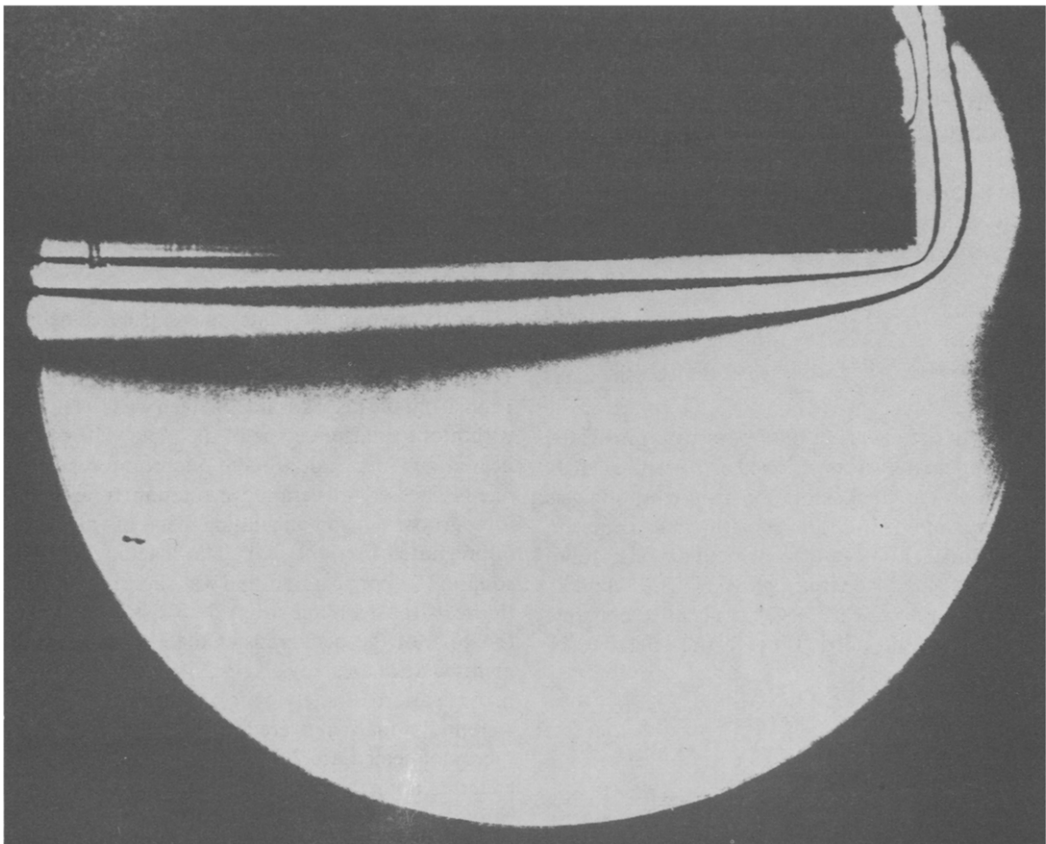


FIG. 3. Infinite fringe interferogram for the 16 cm plate with $\theta_0 = 10$ K.

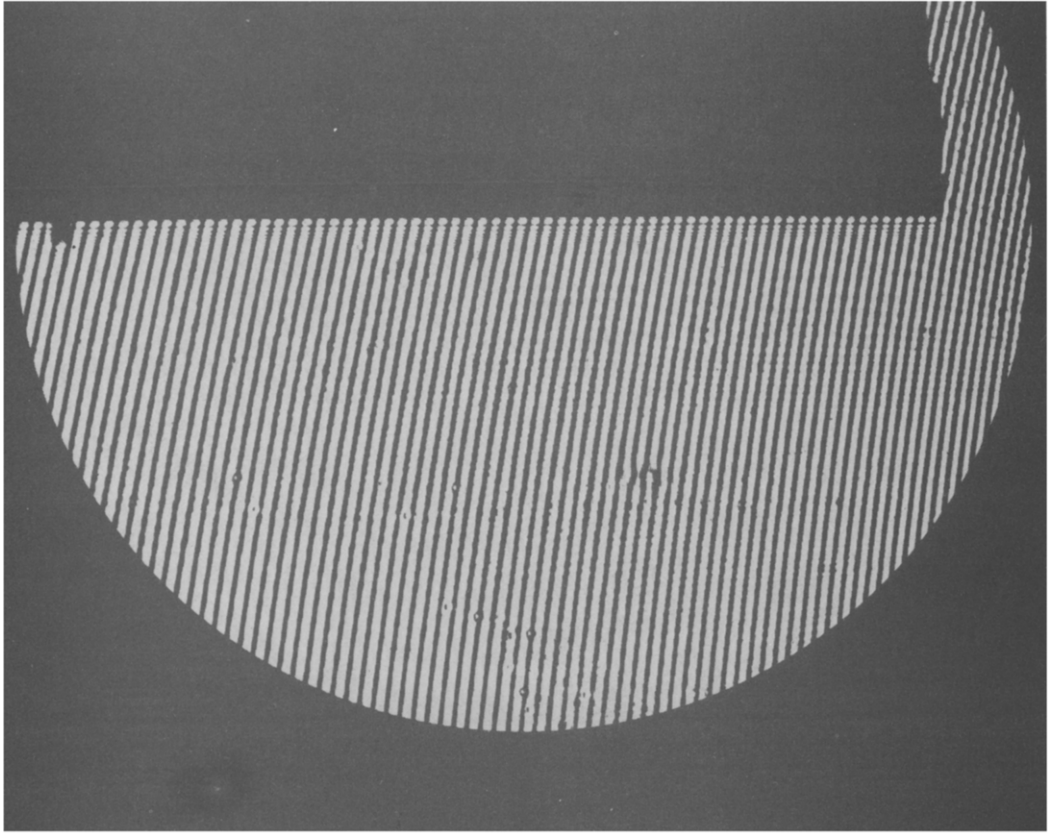


FIG. 4. Finite fringe interferogram for the 16 cm plate, with $\theta_0 = 0$ K.

using Teknol developer (May & Baker Ltd.).

3. DATA ANALYSIS

Raw experimental data were in the form of photographic records of interference patterns. Figure 3 is an interferogram for the 16 cm square plate, with $\theta_0 = 10$ K. This corresponds to the infinite fringe setting of the Mach-Zehnder interferometer and was produced by double-exposure holographic interferometry. The dark interference fringes may be interpreted as isotherms in the air surrounding the test plate. This infinite fringe setting was difficult to achieve experimentally, and an alternate approach was used, the finite fringe setting.

Figure 4 exhibits deliberately introduced finite fringes for the 16 cm plate with $T_0 = T_\infty = 294$ K, i.e. $\theta_0 = 0$ K. The hologram plate was exposed under the prescribed conditions, but rotated very slightly about its vertical axis when re-positioned in its holder. Figure 5 exhibits the interferogram after the plate temperature had been brought to 334 K ($\theta_0 = 40$ K). Adjacent pairs of light or dark fringes result from a change of one wavelength (514.5 nm) in the optical path length of the object beam between the conditions when the hologram was made and the conditions under observation. Differences between the interferograms of Figs. 4 and

5 are due to density changes in the air beneath the plate. Superposition of Figs. 4 and 5, shown schematically in Fig. 6, allows measurement of fringe displacement $\varepsilon(y, z)$ as a function of position, as well as the displacement ε_0 at the plate surface. Figure 7 shows the loci of fringe displacements for Figs. 4 and 5. The procedure for determining fringe displacement was as follows. Prints of the microfilm records were made on Eastman Kodak 4556 Reproduction Film at an enlargement of about three times actual size. Films for Figs. 4 and 5, for example, were superimposed over an illuminated background and values of $\varepsilon(y, z)$ were scaled directly from the films.

Temperature profiles were determined from fringe displacements in the following manner. Since unit fringe displacement results from a change of one wavelength in optical path, the measured displacement is related to the variation in refractive index $n(x, y, z)$ of the air by the equation

$$\varepsilon(y, z) = (2a/\lambda) \int_0^1 dx [n(x, y, z) - n_\infty]$$

in which n_∞ is the refractive index at ambient temperature. According to the Gladstone Dale law, the variation with density ρ of the refractive index of a gas is closely approximated by the equation

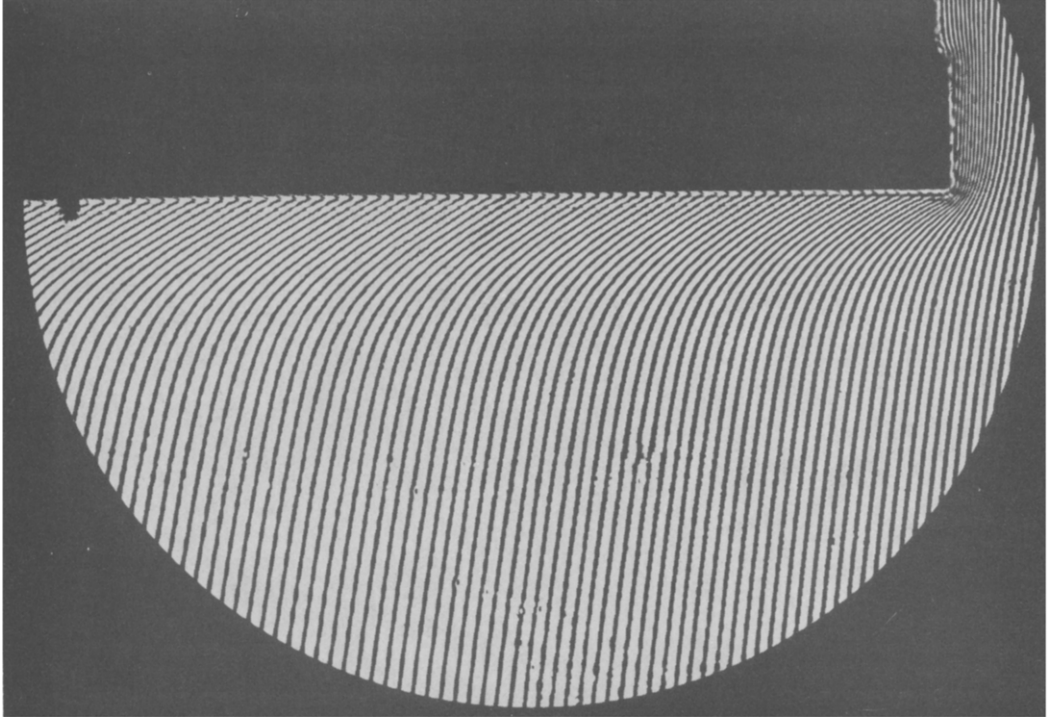


FIG. 5. Finite fringe interferogram for the 16 cm plate, with $\theta_0 = 40$ K.

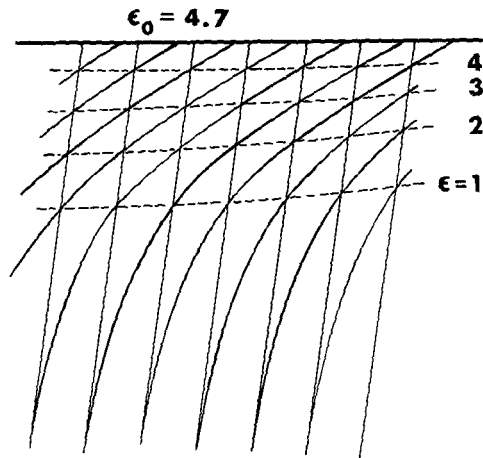


FIG. 6. Superposition of interferograms to determine fringe displacement.

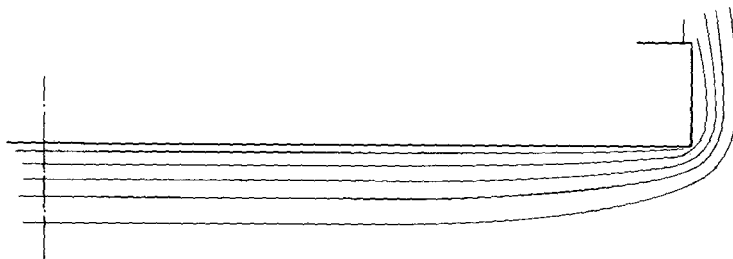


FIG. 7. Loci of fringe displacements for the 16 cm plate, with $\theta_0 = 20$ K.

$$\frac{dn}{dT} = K \frac{d\rho}{dT}$$

Assuming ideal gas behaviour

$$\frac{dn}{dT} = -\frac{Kp}{RT^2} \quad (17)$$

in which p is the ambient pressure, R the ideal-gas constant, and K the Gladstone–Dale constant. Thus

$$n(x, y, z) - n_\infty = -\frac{Kp}{R} \left(\frac{1}{T_\infty} - \frac{1}{T(x, y, z)} \right) \quad (18)$$

and

$$\varepsilon(y, z) = \frac{-2aKp}{R\lambda} \int_0^1 dx \left(\frac{T(x, y, z) - T_\infty}{T_\infty T(x, y, z)} \right). \quad (19)$$

Measured values of $\varepsilon(y, z)$ and ε_0 thus yield a harmonic average temperature, i.e.

$$\frac{\varepsilon(y, z)}{\varepsilon_0} = \int_0^1 dx \left[\frac{T(x, y, z) - T_\infty}{T_0 - T_\infty} \right] \frac{T_0}{T}. \quad (20)$$

However, comparison with theory is facilitated using an average temperature

$$\bar{T}(y, z) = \int_0^1 dx T(x, y, z)$$

or, in dimensionless terms

$$\bar{\phi}(y, z) = \frac{\bar{T}(y, z) - T_\infty}{T_0 - T_\infty} = \int_0^1 dx \phi(x, y, z). \quad (21)$$

Over the entire range of experimental data, the term T_0/T in equation (20) varies only between 1 and 1.17 and $\varepsilon/\varepsilon_0$ is approximately equal to $\bar{\phi}$. A better approximation is to set T equal to \bar{T} in the denominator of equation (20). This leads to the following expression used throughout the data analysis

$$\bar{\phi}(y, z) = \left[1 + \frac{T_0}{T_\infty} \left[\frac{\varepsilon_0}{\varepsilon(y, z)} - 1 \right] \right]^{-1}. \quad (22)$$

For the experimental conditions of this work, the approximation embodied by equation (22) leads to negligible error in $\bar{\phi}$ as z approaches zero, this would not influence evaluation of Nusselt numbers. The approximation is poorest for values of z near the outside of the boundary layer, where T approaches T_∞ . The error is at worst about 8%, for ν near 1 and is less than about 3% for ν less than 0.4.

Although the x -averaged Nusselt number could be determined as

$$\bar{Nu}(y) = -\frac{\partial \bar{\phi}}{\partial z}(y, 0) \quad (23)$$

it was more convenient to determine it as

$$\bar{Nu}(y) = 2 \frac{\partial}{\partial y} [1 - \bar{\phi}^{1/2}]_{y=0} \quad (24)$$

from a graph of $1 - \bar{\phi}^{1/2}$ vs z since that function is expected to be linear in z as z approaches zero, and to have zero intercept. The x -averaged, dimensionless

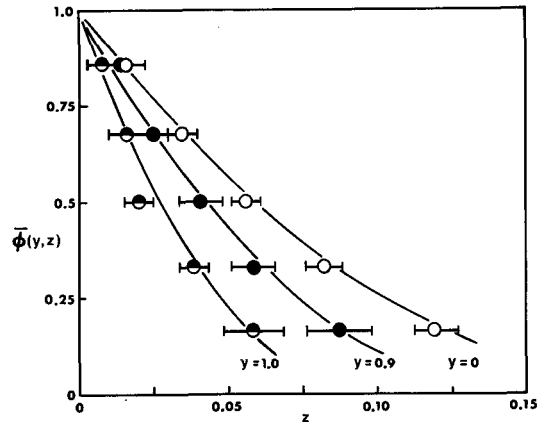


FIG. 8. Air temperature $\bar{\phi} = (T_0 - T_\infty)/(T_0 - T_\infty)$ as a function of distance $z = Z/a$ beneath the 16 cm plate, with $\theta_0 = 20$ K. The solid lines are based on equation (27).

boundary layer thickness is given by equation (7).

4. EXPERIMENTAL RESULTS

For the larger test plate, three different experimental runs were made, each run beginning with the preparation of a hologram of the plate at ambient temperature. For each run, three photographs each were made and analysed for the experimental conditions $\theta_0 = 10, 20, 30$ and 40 K. Thus, nine measurements of fringe displacements, or temperature profiles, were made for each experimental condition. Uncertainties were determined as estimates of the standard deviations of measured positions corresponding to a given fringe displacement or x -averaged air temperature. For the smaller test plate, only one experimental run was made, but three photographs each were made and analysed for the experimental conditions $\theta_0 = 30, 40$ and 50 K.

Figure 8 illustrates typical data for air temperature, in dimensionless form, as a function of dimensionless distance beneath the test plate. Values of $\bar{Nu}(y)$ as a function of Rayleigh number are listed in Table 1. Rayleigh numbers were evaluated at the mean of T_0 and T_∞ . Uncertainties in $\bar{Nu}(y)$ are based on conservative estimates of upper and lower limits in the slope of a plot of $1 - \bar{\phi}^{1/2}$ vs z . Also listed in the table are plate average values, \bar{Nu} . These were determined by trapezoidal integration of $\bar{Nu}(y)$. Uncertainties are conservative estimates based on integrals over the extrema of uncertainties in $\bar{Nu}(y)$.

5. DISCUSSION OF RESULTS

The analytical predictions of Singh, Birkebak, and Drake [8] provide a convenient context within which to discuss experimental results. From equation (11) the predicted temperature profile is

Table 1. Measured values of $\overline{Nu}(y)$ and \overline{Nu} as functions of the Rayleigh number

a, cm	θ_0, K	Ra	$\overline{Nu}(y)$ at $y =$								
			0	0.5	0.6	0.8	0.9	0.95	1.0		
2.55	30	3.94×10^4	5.3 ± 0.2		5.9 ± 0.2	6.3 ± 0.5	7.5 ± 0.5	7.5 ± 0.5			17.8 ± 3.2
	40	4.87×10^4	5.6 ± 0.5		6.3 ± 0.6	7.4 ± 0.9	9.6 ± 1.2	9.6 ± 1.2			17.7 ± 3.8
	50	5.64×10^4	6.0 ± 0.5		6.7 ± 0.6	7.7 ± 0.6	9.4 ± 1.4	9.4 ± 1.4			15.3 ± 3.8
	10	4.88×10^5	7.7 ± 0.4	8.1 ± 0.3		9.6 ± 0.3	11.3 ± 0.2	13.5 ± 0.5			20.1 ± 2.5
8.0	20	8.96×10^5	10.1 ± 0.3	11.2 ± 0.4		12.4 ± 0.5	14.0 ± 0.5	15.9 ± 1.1			21.9 ± 3.0
	30	1.24×10^6	10.8 ± 0.6		12.2 ± 0.5	13.7 ± 0.5	16.0 ± 0.9	16.0 ± 0.9			24.2 ± 3.0
	40	1.53×10^6	12.3 ± 0.6		13.0 ± 0.5	15.5 ± 0.6	18.3 ± 0.7	18.3 ± 0.7			25.2 ± 1.7
			13.5 ± 0.7								

$$\bar{\phi}(y, z) = \int_0^1 dx \phi(x, y, z) = \int_0^{x_{\max}(y)} dx [1 - z/\delta(x, y)]^2 \quad (25)$$

in which $\delta(x, y)$ is given by equation (12), and x_{\max} is the x location at which $z = \delta(x, y)$, i.e.

$$x_{\max} = \{1 - [z/\delta(0, y)]^4\}^{1/2}. \quad (26)$$

In terms of the new variable $v = x/\delta(0, y)$

$$\bar{\phi}(v) = \int_0^{(1-v^4)^{1/2}} dx [1 - v/(1-x^2)^{1/4}]^2 = (1-v^4)^{1/2} + v^2 \sin^{-1}(1-v^4)^{1/2} - 2^{3/2}v[2E(\cos^{-1}v, 2^{-1/2}) - F(\cos^{-1}v, 2^{-1/2})] \quad (27)$$

in which the elliptic integrals are defined as

$$E(\psi, k) = \int_0^\psi dx (1 - k^2 \sin^2 x)^{1/2} \quad (28)$$

$$F(\psi, k) = \int_0^\psi dx (1 - k^2 \sin^2 x)^{-1/2}. \quad (29)$$

In accord with equation (27), measured temperature profiles in the form $\bar{\phi}(v)$ were found to be independent of y . Average values are presented in Table 2 and illustrated in Fig. 9. In preparing the data, values of $\delta(0, y)$ were determined from measured values of $\bar{\delta}(y)$ as described below. Uncertainties are estimates of the standard deviation of values measured at different y locations. As may be seen in the figure, measured temperatures agree well with the predictions of equation (27).

Nusselt numbers may be predicted from equation (27) as follows

$$\overline{Nu}(y) = -\frac{\partial \bar{\phi}}{\partial z}(y, 0) = -\frac{1}{\delta(0, y)} \frac{\partial \bar{\phi}}{\partial v}(0). \quad (30)$$

$$\frac{\partial \bar{\phi}}{\partial v}(0) = -2^{3/2} [2E(\pi/2, 2^{-1/2}) - F(\pi/2, 2^{-1/2})] = -2.396. \quad (31)$$

Thus, making use of equations (10-12)

$$\overline{Nu}(y) = \frac{2.396}{\delta(0, y)} = 0.550(1-y^2)^{-1/4} Ra^{1/5}. \quad (32)$$

From equations (7) and (32)

$$\bar{\delta}(y) = 0.8347 \delta(0, y). \quad (33)$$

The predicted plate average Nusselt number is

$$\overline{\overline{Nu}} = \int_0^1 dy \overline{Nu}(y) = 0.659 Ra^{1/5}. \quad (34)$$

Measured plate average Nusselt numbers, as a function of Rayleigh number are illustrated in Fig. 10 along with the prediction of equation (34). That measured values exceed predicted values is to be expected. The model on which the predictions are based does not account for the finite boundary layer

Table 2. Composite values of $\phi(v)$ averaged over the test plate

a, cm	θ_0, K	Ra	$\phi(v)$	v
2.55	30	3.94×10^4	0.369	0.308 ± 0.008
			0.565	0.205 ± 0.003
			0.767	0.108 ± 0.003
	40	4.87×10^4	0.270	0.393 ± 0.011
			0.413	0.293 ± 0.006
			0.561	0.210 ± 0.004
			0.714	0.138 ± 0.003
			0.874	0.064 ± 0.009
			0.204	0.479 ± 0.009
50	5.63×10^4	0.422	0.296 ± 0.005	
		0.537	0.210 ± 0.013	
		0.657	0.152 ± 0.002	
		0.781	0.093 ± 0.002	
		0.204	0.479 ± 0.009	
		0.422	0.296 ± 0.005	
8.00	10	4.88×10^5	0.330	0.356 ± 0.013
			0.497	0.247 ± 0.003
			0.667	0.147 ± 0.012
			0.838	0.085 ± 0.007
			0.163	0.518 ± 0.017
			0.330	0.348 ± 0.010
	20	8.96×10^5	0.501	0.227 ± 0.024
			0.675	0.147 ± 0.003
			0.854	0.078 ± 0.008
			0.163	0.518 ± 0.017
			0.330	0.348 ± 0.010
			0.501	0.227 ± 0.024
	30	1.24×10^6	0.675	0.147 ± 0.003
			0.854	0.078 ± 0.008
			0.110	0.603 ± 0.027
			0.222	0.438 ± 0.006
			0.336	0.335 ± 0.006
			0.453	0.261 ± 0.010
40	1.53×10^6	0.575	0.199 ± 0.006	
		0.696	0.144 ± 0.003	
		0.822	0.085 ± 0.005	
		0.950	0.026 ± 0.002	
		0.082	0.691 ± 0.021	
		0.166	0.525 ± 0.015	
			0.252	0.429 ± 0.012
			0.340	0.355 ± 0.009
			0.430	0.293 ± 0.008
			0.522	0.235 ± 0.005
			0.617	0.182 ± 0.004
			0.713	0.131 ± 0.004
			0.812	0.085 ± 0.003
			0.914	0.039 ± 0.004

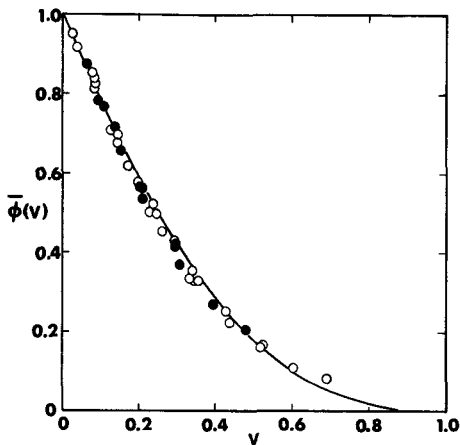


FIG. 9. Composite data for $\bar{\phi}(v)$. The solid line is equation (27). Open circles are for the 16 cm test plate. Closed circles are for the 5.1 cm plate.

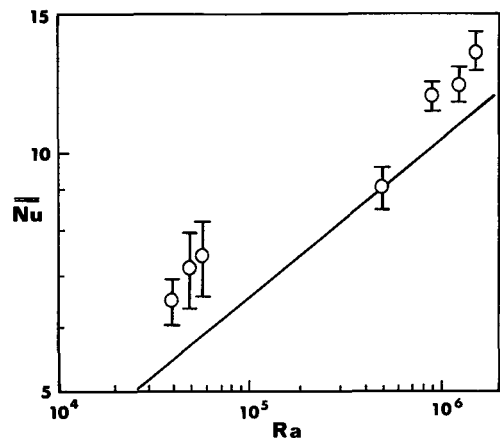


FIG. 10. Plate average Nusselt number as a function of Rayleigh number. The solid line is equation (34).

thickness at the edge of the heat transfer plate.

Uncertainties in measured values are relatively large – 5–10%. These are in part statistical and in part due to the difficulty in measuring the derivative of the temperature profile near the plate surface. Certain systematic errors are included in the uncertainties, namely uncertainties of ± 0.3 K in θ_0 . Other possible systematic errors are not accounted for. These would include refraction errors and errors introduced in fringe displacement by the finite boundary layer thickness at the plate edge. On the basis of the procedures recommended by Goldstein [16], refraction near the plate surface would, at worst, introduce an error $\Delta\epsilon \approx 0.034$. End effects could lead to an error $\Delta\epsilon \approx 0.08$. However, in the procedure used in evaluating temperature profiles, equation (22), only ratios of fringe displacement are involved, and these errors would therefore be minimized.

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MESURE PAR INTERFEROMETRIE HOLOGRAPHIQUE DU TRANSFERT THERMIQUE CONVECTIF SOUS UNE PLAQUE HORIZONTALE, CHAUFFEE DANS L'AIR

Résumé—On utilise la technique de l'interférométrie holographique pour étudier le transfert thermique par convection sous une plaque horizontale, chauffée dans l'air; deux plaques de cuivre carrées sont utilisées avec un côté de 5,1 et 16 cm. Les profils de température sont mesurés sous la surface isotherme à des nombres de Rayleigh variant de 4×10^4 à $1,5 \times 10^6$. L'article décrit l'appareillage et le mode expérimental de la méthode ainsi que le mode d'obtention des profils de température à partir des interférogrammes. Les profils de température obtenus sont en excellent accord avec les solutions analytiques approchées obtenues par d'autres auteurs. Les nombres de Nusselt moyens sur la plaque, obtenus à partir des profils de température, dépassent les valeurs calculées mais sont en accord avec des mesures antérieures.

MESSUNG DES KONVEKTIVEN WÄRMETRANSPORTS UNTERHALB EINER BEHEIZTEN HORIZONTALEN PLATTE IN LUFT DURCH HOLOGRAFISCHE INTERFEROMETRIE

Zusammenfassung — Die Technik der holografischen Interferometrie wurde zur Untersuchung des konvektiven Wärmetransports unterhalb einer beheizten horizontalen Platte in Luft angewandt. Es wurden zwei Kupferplatten von 5,1 und 15 cm im Quadrat verwendet. Temperaturprofile wurden in der Luft unterhalb der isothermen Plattenoberflächen bei Rayleigh-Zahlen im Bereich von 4×10^4 bis $1,5 \times 10^6$ gemessen. Der Aufsatz beschreibt die Apparatur und die Vorgehensweise bei der Anwendung des holografischen Verfahrens, ebenso die Methoden zur Herleitung der Temperaturprofile aus den Interferogrammen. Die gemessenen Temperaturprofile ergaben sich in ausgezeichnete Übereinstimmung mit analytischen Näherungslösungen des Wärmetransportproblems anderer Autoren. Mittlere Nusselt-Zahlen an der Platte, die aus gemessenen Temperaturprofilen abgeleitet wurden, lagen oberhalb der gerechneten Werte, aber stimmten mit früheren Messungen überein.

**ИЗМЕРЕНИЕ МЕТОДОМ ГОЛОГРАФИЧЕСКОЙ ИНТЕРФЕРОМЕТРИИ
КОНВЕКТИВНОГО ТЕПЛОПЕРЕНОСА ПОД НАГРЕТОЙ ГОРИЗОНТАЛЬНОЙ
ПЛАСТИНОЙ В ВОЗДУХЕ**

Аннотация — Голографический интерференционный метод «живых полос» применен для исследования конвективного теплопереноса под нагретой горизонтальной пластиной в воздухе. Были использованы две медные пластины площадью $5,1 \text{ см}^2$, 16 см^2 . Профили температуры измерялись в воздухе под поверхностями изотермических пластин в диапазоне чисел Рэлея от 4×10^4 до $1,5 \times 10^6$. В работе описана установка и процедура голографического измерения, а также методы, использованные для получения температурных профилей из интерферограмм. Измеренные температурные профили продемонстрировали хорошее соответствие с профилями, полученными другими авторами на основе аналитических решений задачи. Усреднённые по пластине числа Нуссельта, полученные из измеренных температурных профилей, превосходят теоретические значения, но согласуются с ранее полученными результатами.